install.packages("expm")

library(expm)

# Problem 7.6 Parts A

Q = matrix(c(-2,1,1,0,1,-3,1,1,2,2,-4,0,1,2,3,-6), nrow=4,byrow=T)

P <- function(t) {expm(t\*Q)}

P(1000)

Output:

[,1] [,2] [,3] [,4]

[1,] 0.4070796 0.3185841 0.2212389 0.05309735

[2,] 0.4070796 0.3185841 0.2212389 0.05309735

[3,] 0.4070796 0.3185841 0.2212389 0.05309735

[4,] 0.4070796 0.3185841 0.2212389 0.05309735

#Part B

P\_0 = diag(4)

# P\_1 is the embedded transition matrix

P\_1 <- matrix (c(0,1/2,1/2,0, 1/3,0,1/3,1/3, 1/2,1/2,0,0, 1/6,1/3,1/2,0), nrow = 4, byrow = T)

for (i in 1:100) {

P\_0 <- P\_0%\*%P\_1

}

P\_0

Output:

[,1] [,2] [,3] [,4]

[1,] 0.2738095 0.3214286 0.297619 0.1071429

[2,] 0.2738095 0.3214286 0.297619 0.1071429

[3,] 0.2738095 0.3214286 0.297619 0.1071429

[4,] 0.2738095 0.3214286 0.297619 0.1071429

# Problem 7.6 Part C

P\_1 = P(1)

P\_1[1,3]

Output: 0.2159691

# Problem 7.6 Part D

P\_4 = P(4)

P\_4[3,1]\*P\_1[3,4]

Output: 0.02146918

# Problem 7.39

lamda <- 15

mu <- 5

#Number of Servers

c <-3

#number of simulations

n <- 1000000

visitor <-0

syst <- rep(0,n)

t <- rep(0,n)

for (j in 1:n) {

arrivalTime <- rexp(1,lamda)

departure <- ifelse(visitor == 0, arrivalTime + 1, rexp(1,mu\*visitor))

change <- ifelse(arrivalTime < departure, 1, -1)

visitor <- visitor + change

syst[j] <- visitor

t[j] <- ifelse(arrivalTime < departure, arrivalTime, departure)

}

syst[1:100]

round(t[1:100],3)

#Total Time

sum(t)

Output: 33352.41

#Average number of visitors in the system

sum(syst)/n

Output: 3.499438